Math 2D Quiz 1 Afternoon - January 14, 2016

Please put name and ID on *both* sides for grading and redistribution.

Show all of your work. [Based on 10.1.14]
*There is a question on the back side.

1. Let $x = e^t - 1$, $y = e^{2t}$ parameterize a curve, where $-\infty < t < \infty$.

3 pts (a) Eliminate the parameter to find a Cartesian equation of the curve and sketch the curve. Indicate, with arrows, the direction in which the curve is traced as the parameter increases. (Hint/Be Careful Graphing: If $-\infty < t < \infty$, what are the ranges of x and y values?)

2 pts (b) Compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. Leave your final answers as functions of t.

a) There's a ~~the~~ few ways to eliminate the parameter. Here's 2 ways.

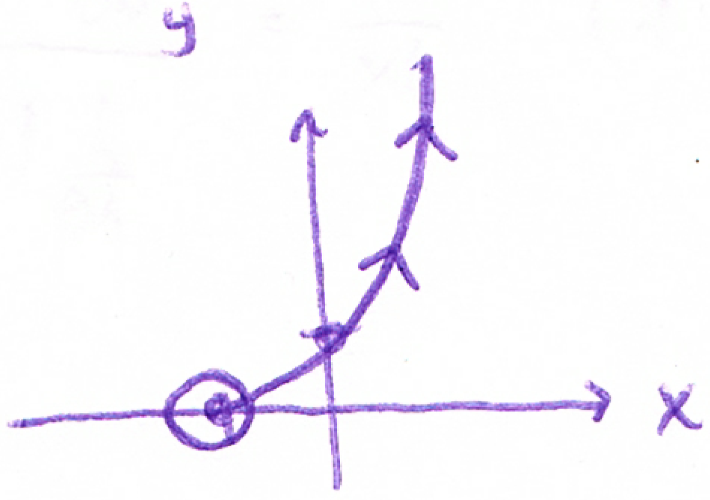
(i) $x = e^t - 1 \Rightarrow e^t = x+1 \Rightarrow \underline{y = e^{2t} = (e^t)^2 = (x+1)^2}$ is the easiest.

(Alternative) ↓

$t = \ln(x+1) \Rightarrow y = e^{2\ln(x+1)} = e^{\ln(x+1)^2} = \underline{(x+1)^2}$ ✓

(ii) $y = e^{2t} \Rightarrow e^t = \sqrt{y} \Rightarrow \underline{x = \sqrt{y} - 1}$

since $-\infty < t < \infty$, e^t is between 0 and ∞ $\Rightarrow$ $x > -1$
$(0 < e^t < \infty)$ $y > 0$

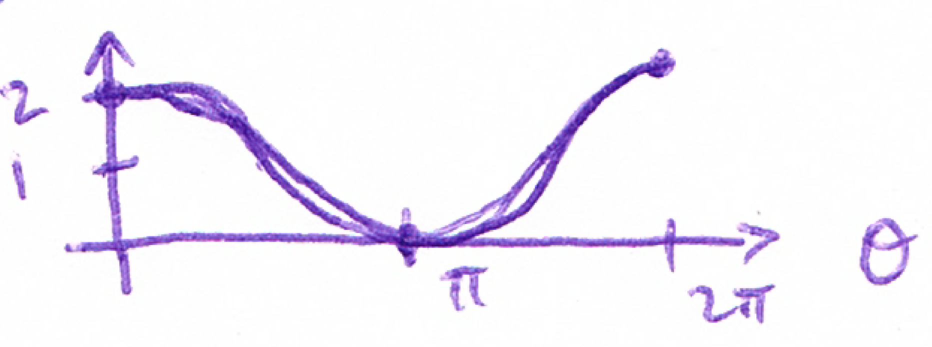


For trajectory, as "$t = -\infty$", we are near $(-1, 0)$
$t = 0$, we are at $(0, 1)$ $\Rightarrow$ Moves upwards.

b) • $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2e^{2t}}{e^t} = \boxed{2e^t}$ [Note: This is $2(x+1)$, "$\frac{dy}{dx} = 2(x+1)$" is what we expect w/ $y = (x+1)^2$]

• $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{2e^t}{e^t} = \boxed{2}$ [Again, consistent w/ $y = (x+1)^2$].

✲ Some people graphed $r = 1+\cos\theta$ Like "$y = 1+\cos x$"



instead of tabulating points.

[Based on 10.3.63]

2. Let $r = 1 + \cos\theta$.

2 pts (a) Sketch the curve.

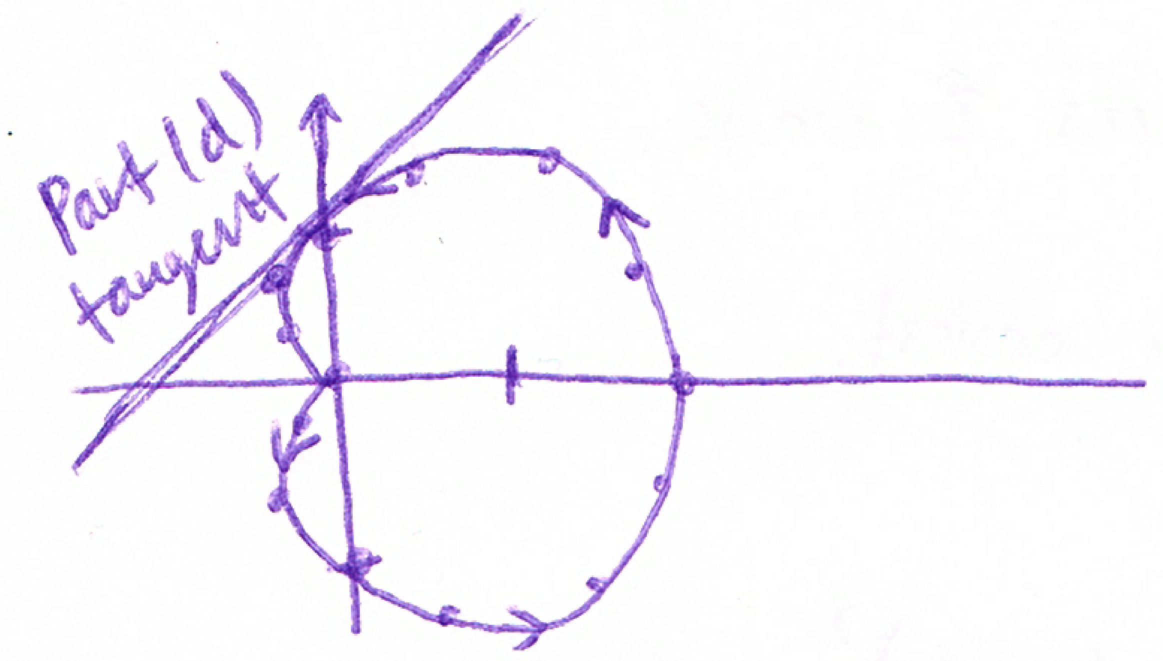
2 pts (b) Compute $\frac{dy}{dx}$ as a function of θ.

1 pt (c) When $\theta = \pi/2$: What is the (x, y) coordinate on the curve? What is the value of $\frac{dy}{dx}$?

(d) Using part (c), write the equation of the tangent line to the curve when $\theta = \pi/2$.

a)

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
r	2	$1+\frac{\sqrt{3}}{2}$	$1+\frac{\sqrt{2}}{2}$	$\frac{3}{2}$	1	$\frac{1}{2}$	$1-\frac{\sqrt{2}}{2}$	$1-\frac{\sqrt{3}}{2}$	0	$1-\frac{\sqrt{3}}{2}$	$1-\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$3/2$	$1+\frac{\sqrt{2}}{2}$	$1+\frac{\sqrt{3}}{2}$	2



✲ A shortcut is to note $\cos\theta$ is even.

✲ Recall: $x = r\cos\theta$, $y = r\sin\theta$ // plug in $r = 1+\cos\theta$.

b) $$\frac{dy}{dx} = \frac{\frac{d}{d\theta}(r\sin\theta)}{\frac{d}{d\theta}(r\cos\theta)} = \frac{\frac{d}{d\theta}(\sin\theta + \sin\theta\cos\theta)}{\frac{d}{d\theta}(\cos\theta + \cos^2\theta)} = \boxed{\frac{\cos\theta + \cos^2\theta - \sin^2\theta}{-\sin\theta - 2\cos\theta\sin\theta}}$$

This also equals $\frac{\cos\theta + \cos 2\theta}{-\sin\theta - \sin 2\theta}$.

c) At $\theta = \frac{\pi}{2}$:

- $x = r\cos\theta = 0$
- $y = r\sin\theta = 1 \cdot \sin\frac{\pi}{2} = 1$. $\rightarrow$ $\boxed{(x,y) = (0,1)}$
- $\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = \frac{\cancel{\cos\pi/2} + \cancel{\cos^2\pi/2} - \sin^2\pi/2}{-\sin\pi/2 - 2\cancel{\cos\pi/2}\sin\pi/2} = \frac{-1}{-1} = \boxed{1}$

d) Using (c), it's $\underline{y - 1 = x}$ $\underline{(y = x+1)}$. It's appropriate on the graph.